Teaching Mathematics with Technology

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Introduction: what we will, and will NOT talk about!

- Teaching/Learning in an advanced technological society has multiple aspects: The Web, Smart Classrooms, Calculators and Computers, and … AI (Artificial Intelligence)!

- Teaching Mathematics has levels of tech involved:
  0) No calculators allowed! \( \frac{1}{2} + \frac{1}{3} = ? \ldots \) by hand
  1) TI-83 for numerical computations;
  2) TI-89 for symbolic computations \( \frac{d}{dx}(\sin(x))=? \ldots \)
  3) Math Soft: GeoGebra, CoCalc (SAGE), Mathematica etc.
  4) **AI:** the analog of Alexa & Siri for Mathematics;
     (… less the annoying voice recognition issue ;)
  5) Brain implants (Calculus tattoo ;) and avatars … ]
Tools and Machines Assisting Teaching / Learning

- What we use is conditioned by our goals: what type of students we prepare … (long story …);
- Math Software provide a framework for creating: solutions for the homework assignments, having editing capability, graphical interface and computational capabilities;
- Case Study Goal: “Using AI-Mathematica to improve the delivery of conceptual aspects in MAT 147 Calculus III”.

… what triggered this initiative? A) MAT 200 new course
What triggered this initiative?

A) The need to rise the conceptual interface in general mathematics courses, to compensate the exponential growth of knowledge, which in turn demanded:

B) Preparing a new course:

   **MAT 200 Teaching Mathematics using with Technology**

and the symptomatic fact:

C) Without using Math Software, Calc III content delivery usually stops at the beginning of Ch. 16: Green / Stokes / Gauss Theorems (**Fundamental Theorems of Vector Calculus**; all the “goodies” needed in Sciences, especially in Physics).
What is “AI-Math”? 
The Teaching / Learning Process

- Teaching “flow”:

  Teacher → Programmer

- Example:

  Left) Calc III Teacher, with Stewart’s “Calculus with Early Transcendentals” as Textbook, use a free-form interface to Wolfram Mathematica (Math Software), to “hide” the computational aspects and teach the Student the conceptual aspects.

  Right) The Teacher instructs the Programmer, to develop a Calculus Grammar, to “Teach” Wolfram Mathematica to understand “Stewart’s Calculus”. The Student “talks” to Al-Math ...
Concrete Example: Ch. 16.8 Problem 13

“Verify that Stokes’ Theorem is true for the given vector field and surface”:

13. \( \mathbf{F}(x, y, z) = y^2 \mathbf{i} + x \mathbf{j} + z^2 \mathbf{k} \),

\( S \) is the part of the paraboloid \( z = x^2 + y^2 \) that lies below the plane \( z = 1 \), oriented upward

Remarks:
1) Note the “verbs” and “nouns” for this grammar.
2) The initial stages of the pedagogic process will train the student to do this by hand! (computing the line/surface integral).
Sample of AI-Math Code (*freeform_stokes.pdf*)

- Sample of what goes back-and-forth between the Teacher & Programmer … (No need for You / Student to know :)
  
  A) Textbook info, calculus jargon, grammar …
  
  B) Mathematica implementation of freeform … etc.

  C) Outputs: Pictures and Solutions to be used by Teacher to train the Student: Calculus language & AI-Mathematica computations.

- Note: is is like talking to Alexa and Siri Calculus, but without the voice recognition glitches; the Student needs to know the correct “pronunciation” of Calculus terms and grammar!
Simpler Example: a Line Integral

- The **Student** uses the concepts to **command AI-Math**, and gets the **Solution** steps (needs to understand the parts).
- The Student further **contributes to the Solution**, explaining it in writing to the Teacher.
The “Big Picture”

Who’s Teaching Who!?

[Teacher/Student/Programmer/AI: are all involved!]
Technology Impacts Education

- We live in a Technological Society; general framework for T&L is affected by this (3-simplex interactions bi-directional arrows: T/L is a duplex process):

  Teacher  Programmer
  Student   AI-Mathematics

- Sample of current uses of Tech in Teaching: from TI-83/89 & GeoGebra, to SAGE & Wolfram Mathematica (goals have been discussed by other presenters).

- The impact of AI in Teaching/Learning and Main Goal: rise the level of the Conceptual Interface to Math [e.g. be able to Teach Ch.16 in Calc 3 thoroughly].

- The specific project of the author: add an AI-Interface to Wolfram Mathematica for use in Calc III (freeform input / grammar to be generalized and used in Abstract Algebra, Complex Analysis etc.).
Roles of Technology in Mathematics Education
(from the article by C.J. Cullen, J.T. Hartel and M. Nickels)

- Besides A) the role of technology in delivering curriculum, other four prominent theme were identified (loc. cit.):
  B) Promoting cycles of proof (explore-> conjecture->test & revise ->prove; promotes the Scientific R&D method);
  C) Presenting and connecting multiple representations;
  D) Supporting case-by-case reasoning (simulating / analyzing / organizing DATA);
  E) Serving as a Tutee (Programmer “tutors” the Machine).

- The use of AI-Math in Math-Ed, as proposed here, contributes to all of these “themes” ...
Technology and Artificial Intelligence: Enhancing and Refining the Educational Process

A) Technology *accelerates and enhances curriculum delivery* (while Sciences deliver mostly 1500 and later developments, K-12 Math delivers mainly ancient Egyptian and Greek curriculum);

B) Allows students *practice the scientific method* (Math model / conjecture, experiment/test, confirm/prove);

C) Tech enhances the graphical, pictorial and diagrammatic representation of concepts and methods (Compare Windows WYSIWYG vs. UNIX: line interpreter, typical modality for Abs. Algebra);

D) Capable of fast generation of examples, scenarios, problems;

E) The *roles of Teaching and Learning are distributed* among Teacher, Student, Programmer, Machine (Calculator/AI).
Teaching and Learning as Duplex Process

- *Teaching has a learning component* and learning provides feedback, containing a teaching component (Duplex Channel).
- Recall the parties involved:

```
Teacher  →  Programmer
        ↓       ↓
Student  ↓  AI-Math
```

... every arrow is in fact a weighted duplex channel (feedback).
On Textbooks, WebAssign, Math-Software and AI

- **WebAssign**-like environments provide a limited teaching experience for completing Homework: lack of creating a documentation of the solution;

- **Textbooks** provide the standard Math-language, but do not bridge the gap to Math-Software;

- **The solution**: design *Mathematica* “plug-ins” (like in a browser), playing the *role of adapters to specific Textbooks*. These adapters/drivers add an AI-component to software (Freeform I/O-interface; based on Grammars: Theory of Formal Languages, a.k.a. Automata Theory; iconic level interface available).
Everybody Gets Promoted (Win-Win situation)

- In this process of developing AI, the Machines will not take over (The Matrix / Terminator scenarios):
  A) In **Agriculture**, smart combines, are self-driving, taught by us …
  B) In **Applied Mathematics**, AI-Math allows to focus on **Math-Modeling & Science Applications** …

- The distinction between **Teacher and Programmer** blurs, and **Student-Machine becomes a Teachable UNIT**: we teach students teach machines, that will work for us …
  (... and we’ll just go fishing more often :).
Alice, Alexa, Siri and Jarvis: our friendly workforce!

[Some fun stuff, maybe …]

- Alice the chatbot

- Talking to AI-Math feels like Tony Stark talking to Jarvis (the CAD part); … amazing interfaces!

- Outcome: students will relate to MODERN Tech-Teaching

[Calculus per se is 400 years old ;]
Credits / The Cast in this Math-ED R&D

- **Teacher**: Lucian M. Ionescu, Math. Dept, ISU (US)
- **Programmer**: Dara Shaida, Ireland (IE), working for:
- **AI-Software Development Co.**: Computational Class Notes (CCN), based in Ireland, which is a partner of:
- **Math Software**: Wolfram Research (Champaign, IL).

… made possible by that thing called “Internet” …
Supplements
What “make” of students we envision?

- Modern teaching of General Mathematic Courses aims to prepare the Student to become an “Application Oriented Mathematician”:
  - Using “Standard” Mathematica (Mathematical software) to assist the Math-Modeling, performing the computations, representing graphically, producing the associated documentation.
  - Using an AI-Math Interface: the Student-Machine Team is similar to how Stark from Iron Man talks to Jarvis to perform the needed tasks.
- Currently I collaborate with CCN (Wolfram Licenced Developer) to build such a “Math-Jarvis” interface to Wolfram Mathematica, as an “adapter”/plug-in/DRIVER between a Math Textbook (e.g. Stewart Calculus & Brown/Churchill Complex Analysis and Applications), and Wolfram Mathematica ver. 12.
- As an specific example of benefit: Calc III instructors will be able to teach Ch. 16 (General Stokes Theorem), which is the core/target result of Vector Calculus. [Earlier in the talk: why computations get in the way of teaching.]
Machine Learning and Evolving Software (AlphaGo)

- “Standard” software tools for writing: Word, research papers: LaTeX, statistics R etc., and now for Mathematics Teaching and Homework: Mathematica.
- Mathematica can go the distance all the way to top abstract / modern research, e.g. Renormalization in QFT: a key structure is the Hopf algebra of rooted trees, together with Feynman Rules for setting-up Feynman Integrals for F. Diagrams. It can be implemented using custom gramers, as “free-form drivers” of Mathematica [a driver is like an adapter: a change of interface, a translator etc.]
- This is usually done by specialized developers / programmers, e.g. CCN.
- The NEXT Step (currently under implementation) is to have an AI-Module, which does that: EVOLVES Mathematica Free-Form Interface (See how AlphaGo learned Go); this is a typical use of Machine Learning, using Neural Networks [this is NOT just a research topic: it’s deeply production related/involved; see 1, 2.]
- A lot for Math-Ed to learn from CS; eben then, teaching AI will be more productive than teaching students traditional Arts & Sci; … more time to go fishing!
“LossOfGenerality.com” is a **Non-profit org**: Education on Demand.

- MI-Modules (Math-Intelligent): textbook substitutes for how-to, at the follow-up chapters, i.e. in 16.7 they learn hands-on Surface Integrals, relying on 16.6 where they’ve learned hands-on computing double integrals; but now at 16.7, they learn the MI-module for accessing M for that.

- Conceptual Hierarchical Flow Charts are a must (knowledge structure);

- Teaching students How2Learn is a must: how knowledge is built and what is made of …

- We use to teach student’s brains Mathematics; now we teach them to be programmers of various “Computational Machines” (learning what brands exit, they interface and how to “teach” them …); and then “Thinking Machines” (AI in various Sciences, e.g. MI-modules).
Wolfram Research - Web Associated Resources

- **Wolfram Cloud**: allows to have all your “Math-stuff” there …
- Nice tutorials and documentations, e.g. Primer of Wolfram Language:
  
  5 min & More (at the end of 5 min).
- **Things to Try with a Notebook**; at the end has the links for MORE;
- **Using Notebooks**
- **Wolfram Extension Package**: what makes grammars implementable;
## Introduction to Complex Analysis MAT 349

Code: ISUW20complex

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CCN Cloud Access - Example: L. M. Ionescu