Teaching Mathematics with Technology

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Introduction: what we will, and will NOT talk about!

- Teaching/Learning in an advanced technological society has multiple aspects: The Web, Smart Classrooms, Calculators and Computers, and … AI (Artificial Intelligence)!
- Teaching Mathematics has levels of tech involved:
    0) No calculators allowed! [½ + ⅓ = ? … by hand]
    1) TI-83 for numerical computations;
    2) TI-89 for symbolic computations [d/dx(sin(x))=? …]
    3) Math Soft: GeoGebra, CoCalc (SAGE), Mathematica etc.
    4) AI: *the analog of Alexa & Siri for Mathematics*;
        (... less the annoying voice recognition issue ;)
    [5) Brain implants (Calculus tattoo ;) and avatars …]

Tools and Machines Assisting Teaching / Learning

- What we use is conditioned by our goals: what *type of students* we prepare … (long story …);
- Math Software provide a *framework for creating*: solutions for the homework assignments, having **editing** capability, **graphical** interface and **computational** capabilities;
- Case Study Goal: "*Using AI-Mathematica to improve the delivery of conceptual aspects in MAT 147 Calculus III*".

… what triggered this initiative? A) MAT 200 new course

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A) The need to rise the *conceptual interface* in general mathematics courses, to compensate the exponential growth of knowledge, which in turn demanded:

B) Preparing a new course:

MAT 200 Teaching Mathematics using with Technology

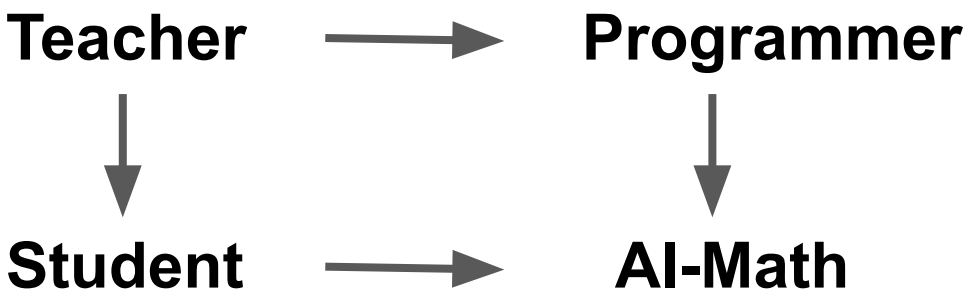
and the symptomatic fact:

C) Without using Math Software, Calc III content delivery usually stops at the beginning of Ch. 16: Green / Stokes / Gauss Theorems (***Fundamental Theorems of Vector Calculus***; all the “goodies” needed in Sciences, especially in Physics).

What is “AI-Math”?

The Teaching / Learning Process

- Teaching “flow”:



- Example:

Left) Calc III **Teacher**, with Stewart’s “Calculus with Early Transcendentals” as **Textbook**, use a free-form interface to Wolfram Mathematica (Math Software), to “hide” the computational aspects and teach the **Student** the conceptual aspects.
Right) The Teacher instructs the **Programmer**, to develop a Calculus Grammer, to *”Teach” Wolfram Mathematica to understand “Stewart’s Calculus”*. The Student “talks” to AI-Math ...

Concrete Example: Ch. 16.8 Problem 13

"Verify that **Stokes' Theorem** *is true* for the given **vector field** and **surface**":

13. $\mathbf{F}(x, y, z) = y^2\,\mathbf{i} + x\,\mathbf{j} + z^2\,\mathbf{k}$,
S is the part of the paraboloid $z = x^2 + y^2$ that lies below the plane $z = 1$, oriented upward

Remarks:

1) Note the "verbs" and "nouns" for this grammar.

2) The initial stages of the pedagogic process will train the student to do this by hand! (computing the line/surface integral).

Sample of AI-Math Code (freeform_stokes.pdf)

- Sample of what goes back-and-forth between the Teacher & Programmer … (No need for You / Student to know :):

    A) Textbook info, calculus jargon, grammar …

    B) Mathematica implementation of freeform … etc.

    C) Outputs: Pictures and Solutions to be used by Teacher to train the Student: Calculus language & AI-Mathematica computations.

- Note: is is like talking to Alexa and Siri Calculus, but without the voice recognition glitches; the Student needs to know the correct “pronunciation” of Calculus terms and grammar!

Simpler Example: a Line Integral

In[14]:= "find the clockwise line integral of
F(x,y,z)=(-y^2,x,z^2) along the intersection of y+z=2 and x^2+y^2=1"

In[15]:= "steps" ¿

Stokes' Theorem:

$$\iint_S F \cdot dS = \int\int_D \left(-P\frac{\partial g}{\partial x} - Q\frac{\partial g}{\partial y} + R\right) dA$$

Step 1:

Compute the Curl

$$\nabla (-y^2)\,\vec{i} + (x)\,\vec{j} + (z^2)\,\vec{k} = (0)\,\vec{i} + (0)\,\vec{j} + (2y+1)\,\vec{k}$$

Step 2:

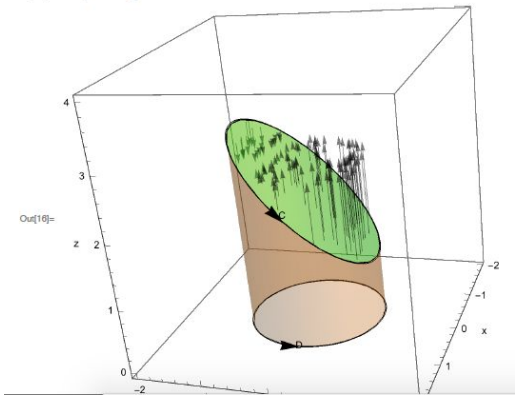
Form the Cartesian coordinates integrals

$$\int\int_D \left(-P\frac{\partial g}{\partial x} - Q\frac{\partial g}{\partial y} + R\right) dA$$

$$= -1 \times \int_0^{2\pi}\int_0^1 (1+2y)\, dA$$

In[16]:= "Splot" ¿

Out[16]=



- The ***Student*** uses the concepts to *command AI-Math*, and gets the ***Solution*** steps (needs to understand the parts).
- The Student further *contributes to the Solution*, explaining it in writing to the Teacher.

The “Big Picture”

Who’s Teaching Who!?

[Teacher/Student/Programmer/AI: are all involved!]

Technology Impacts Education

- We live in a Technological Society; general framework for T&L is affected by this (3-simplex interactions bi-directional arrows: T/L is a duplex process):

Teacher	Programmer
Student	AI-Mathematics

- Sample of current uses of Tech in Teaching: from TI-83/89 & GeoGebra, to SAGE & Wolfram Mathematica (goals have been discussed by other presenters).

- The impact of AI in Teaching/Learning and Main Goal: rise the level of the **Conceptual Interface to Math** [e.g. be able to Teach Ch.16 in Calc 3 thoroughly].

- The specific project of the author: add an AI-Interface to Wolfram Mathematica for use in Calc III (freeform input / grammar to be generalized and used in Abstract Algebra, Complex Analysis etc.).

Roles of Technology in Mathematics Education

(from the article by C.J. Cullen, J.T. Hartel and M. Nickels)

- Besides A) the role of technology in delivering curriculum, other four prominent theme were identified (loc. cit.):

  B) Promoting **cycles of proof** (explore-> conjecture->test & revise ->prove; promotes the Scientific R&D method);

  C) Presenting and connecting **multiple representations**;

  D) Supporting case-by-case reasoning (simulating / analyzing / organizing DATA);

  E) Serving as a Tutee (Programmer "tutors" the Machine).

- The use of AI-Math in Math-Ed, as proposed here, contributes to all of these "themes" ...

Technology and Artificial Intelligence: Enhancing and Refining the Educational Process

A) Technology *accelerates and enhances curriculum delivery* (while Sciences deliver mostly 1500 and later developments, K-12 Math delivers mainly ancient Egyptian and Greek curriculum);

B) Allows students *practice the scientific method* (Math model / conjecture, experiment/test, confirm/prove);

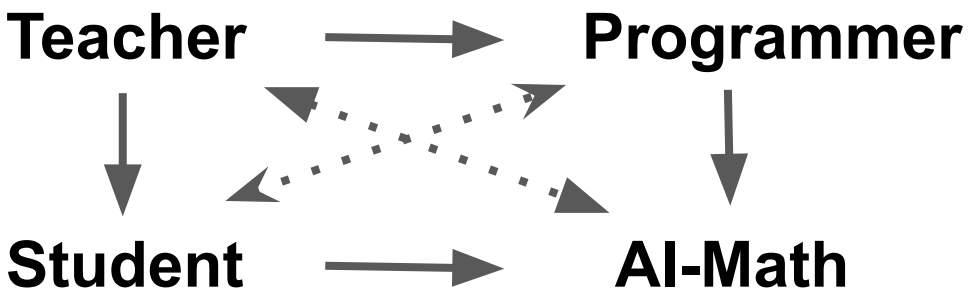
C) Tech enhances the graphical, pictorial and diagrammatic representation of concepts and methods (Compare Windows WYSIWYG vs. UNIX: line interpreter, typical modality for Abs. Algebra);

D) Capable of fast generation of examples, scenarios, problems;

E) The roles of Teaching and Learning are distributed among *Teacher, Student, Programmer, Machine* (Calculator/AI).

Teaching and Learning as Duplex Process

- *Teaching has a learning component* and learning provides feedback, containing a teaching component (Duplex Channel).
- Recall the parties involved:



... every arrow is in fact a weighted duplex channel (feedback).

On Textbooks, WebAssign, Math-Software and AI

- WebAssign-like environments provide a limited teaching experience for completing Homework: lack of creating a documentation of the solution;
- Textbooks provide the standard Math-language, but do not bridge the gap to Math-Software;
- The solution: design *Mathematica "plug-ins"* (like in a browser), playing the *role of adapters to specific Textbooks*. These adapters/drivers add an AI-component to software (Freeform I/O-interface; based on Grammars: Theory of Formal Languages, a.k.a. Automata Theory; iconic level interface available).

Everybody Gets Promoted (Win-Win situation)

- In this process of developing AI, the Machines will not take over (The Matrix / Terminator scenarios):

A) In **Agriculture**, smart combines, are self-driving, taught by us …

B) In **Applied Mathematics**, AI-Math allows to focus on *Math-Modeling* & *Science Applications* ...

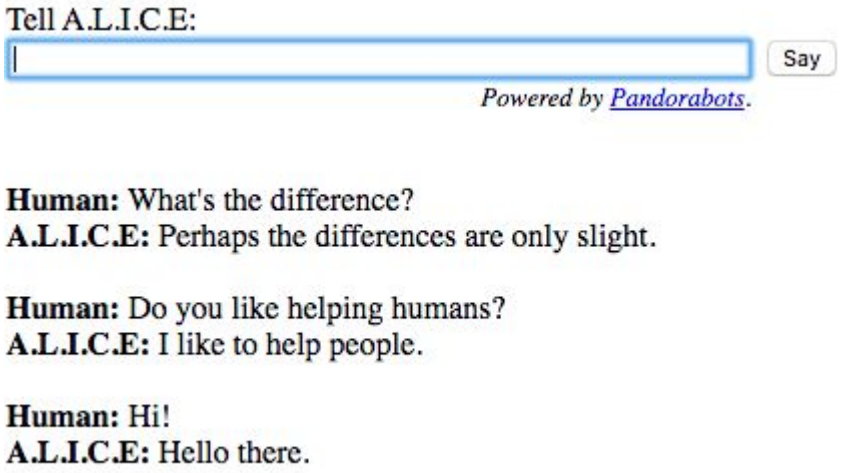
- The distinction between *Teacher and Programmer* blurs, and *Student-Machine becomes a Teachable UNIT*: we teach students teach machines, that will work for us …

(... and we'll just go fishing more often :).

Alice, Alexa, Siri and Jarvis: our friendly workforce!

[Some fun stuff, maybe …]

- Alice the chatbot



- Talking to AI-Math feels like Tony Stark talking to Jarvis (the CAD part);
… amazing interfaces!

- Outcome: *students will relate to MODERN Tech-Teaching*
[Calculus per se is 400 years old ;]

Credits / The Cast in this Math-ED R&D

- Teacher: Lucian M. Ionescu, Math. Dept, ISU (US)
- Programmer: Dara Shaida, Ireland (IE), working for:
- AI-Software Development Co.: Computational Class Notes (CCN), based in Ireland, which is a partner of:
- Math Software: Wolfram Research (Champaign, IL).

… made possible by that thing called “Internet” ...

Supplements

What “make” of students we envision?

- Modern teaching of General Mathematic Courses aims to prepare the Student to become an “Application Oriented Mathematician”:
  - Using “Standard” Mathematica (Mathematical software) to assist the Math-Modeling, performing the computations, representing graphically, producing the associated documentation.
  - Using an AI-Math Interface: the Student-Machine Team is similar to how Stark from Iron Man talks to Jarvis to perform the needed tasks.
- Currently I collaborate with CCN (Wolfram Licenced Developer) to build such a “Math-Jarvis” interface to Wolfram Mathematica, as an “adapter”/plug-in/DRIVER between a Math Textbook (e.g. Stewart Calculus & Brown/Churchill Complex Analysis and Applications), and Wolfram Mathematica ver. 12.
- As an specific example of benefit: Calc III instructors will be able to teach Ch. 16 (General Stokes Theorem), which is the core/target result of Vector Calculus. [Earlier in the talk: why computations get in the way of teaching.]

Machine Learning and Evolving Software (AlphaGo)

- “Standard” software tools for writing: Word, research papers: LaTeX, statistics R etc., and now for Mathematics Teaching and Homework: Mathematica.
- Mathematica can go the distance all the way to top abstract / modern research, e.g. Renormalization in QFT: a key structure is the Hopf algebra of rooted trees, together with Feynman Rules for setting-up Feynman Integrals for F. Diagrams. It can be implemented using custom gramers, as “free-form drivers” of Mathematica [a driver is like an adapter: a change of interface, a translator etc.]
- This is usually done by specialized developers / programmers, e.g. CCN.
- The NEXT Step (currently under implementation) is to have an AI-Module, which does that: EVOLVES Mathematica Free-Form Interface (See how AlphaGo learned Go); this is a typical use of Machine Learning, using Neural Networks [this is NOT just a research topic: it’s deeply production related/involved; see 1, 2.].
- A lot for Math-Ed to learn from CS; eben then, teaching AI will be more productive than teaching students traditional Arts & Sci; … more time to go fishing!

Dara Project - Mathematical Intelligence / Assistant

- “LossOfGenerality.com” is a Non-profit org: Education on Demand.

- MI-Modules (Math-Intelligent): textbook substitutes for how-to, at the follow-up chapters, i.e. in 16.7 they learn hands-on Surface Integrals, relying on 16.6 where they’ve learned hands-on computing double integrals; but now at 16.7, they learn the MI-module for accessing M for that.

- Conceptual Hierarchical Flow Charts are a must (knowledge structure);

- Teaching students How2Learn is a must: how knowledge is built and what is made of …

- We use to teach student’s brains Mathematics; now we teach them to be programmers of various “Computational Machines” (learning what brands exit, they interface and how to “teach” them …); and then “Thinking Machines” (AI in various Sciences, e.g. MI-modules).

Wolfram Research - Web Associated Resources

- Wolfram Cloud: allows to have all your “Math-stuff” there …
- Nice tutorials and documentations, e.g. Primer of Wolfram Language:

  5 min & More (at the end of 5 min).
- Things to Try with a Notebook; at the end has the links for MORE;
- Using Notebooks
- Wolfram Extension Package: what makes grammars implementable;

CCN Cloud Access - Example: L. M. Ionescu